Mycielski among trees - category case

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The 2-dimensional version of classical Mycielski Theorem says that every comeager or conull subset of $[0,1]^2$ contains a square of a form $P \times P$, where P is a perfect subset of $[0,1]$. We consider generalizations of this theorem by replacing a perfect square with a rectangle of a form $[T_1] \times [T_2]$, where $T_1 \subseteq T_2$ are some type of trees (perfect, uniformly perfect, Silver, Miller or Laver) and $[T]$ denotes a body of a tree T.

During this talk we will focus on category case. In particular we will show that for every comeager G_δ set $G \subseteq \omega^\omega \times \omega^\omega$ there exists a Miller tree $M \subseteq \omega^{<\omega}$ and a uniformly perfect tree $P \subseteq M$ such that $[P] \times [T] \subseteq G$. We will also show that this result is somehow optimal - we cannot replace P with a Silver tree or a Miller tree and no comeager G_δ contains a square of bodies of Silver trees or Laver trees.

These results come from a joint work with Robert Rałowski and Szymon Żeberski.